TRANSIENT THERMAL STRESSES IN HOLLOW CIRCULAR CYLINDERS

F. W. TEEPE

THESIS
T227

Library
U. S. Naval Postgraduate School
Monterey, California

211
B

TRANSIENT THERMAL STRESSES
IN
HOLLOW CIRCULAR CYLINDERS

By

Frederick W. Teepe
Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE

United States Naval Postgraduate School

Monterey, California

1 9 5 2

Thesis
T227

TRANSIENT THERMAL STRESSES

in

HOLLOW CIRCULAR CYLINDERS

* * *

F. W. Teepe

18062

This work is accepted as fulfilling
the thesis requirements for the degree of
Master of Science
in
Mechanical Engineering
from the
United States Naval Postgraduate School.

Chairman
Department of Mechanical Engineering

Approved:

Academic Dean

ACKNOWLEDGEMENT

The author wishes to express his appreciation for the willing assistance given him by the Mechanical Engineering Department of the United States Postgraduate School. Special acknowledgement is due Professor D. E. Drucker, whose continued guidance and advice have been of immeasurable help, as has been the aid offered by Professor R. E. Newton.

ACKNOWLEDGMENT

The author wishes to express his appreciation for the willing assistance given him by the Mechanical Engineering Department of the United States Postgraduate School. Special acknowledgement is due Professor [illegible], whose continual guidance and advice have been of inestimable help, as has been the aid offered by Professor [illegible].

LIST OF ILLUSTRATIONS

FIGURE NUMBER	TITLE
A.	Curves of surface temperature vs. time (Analytic and numerical solutions)
B.	Curves of solid temperature vs. distance (Analytic and numerical solutions)
C.	Curves of surface temperature vs. time for various values of N (Numerical solution)
D.	Curves of surface temperature vs. time for various values of N (Numerical solution - Log-log plot)
E.	Curves of temperature times radius vs. radius (Analytic solution)
F.	Curves of stress vs. radius at time of maximum stress (Analytic solution)
G.	Curves of maximum stress vs. time (Analytic solution)

TABLE OF SYMBOLS

Numerical Method (temperature-time determination)

c_p	Specific heat at constant pressure BTU/#, °F
F_{oo}, F_{ol}, etc.	Temperature influence coefficients
H	Surface heat transfer coefficient BTU/ft^2, hr, °F
k	Thermal conductivity BTU/hr, ft^2, °F/ft
ρ	Density in #/ft^3
Δt	Finite arbitrary incrument of time hrs
T_1, T_2, etc.	Temperature of successive elemental blocks °F
T_2'	Temperature of elemental block at $(t + \Delta t)$ °F
$K = k/c_p \rho$	Thermal diffusivity ft^2/hr
$M = \frac{c_p \Delta x^2 \rho}{k\ t} = \frac{\Delta x^2}{K\ t}$	Dimensionless modulus
$N = \frac{h\ \Delta x}{k}$	Surface modulus
T_o	Initial temperature of solid °F
T_a	Temperature of heating medium °F

Analytic Method (temperature-time determinations)

c_p	Specific heat at constant pressure BTU/#, °F
e	Base of natural logarithms - 2.718....
H	Surface heat transfer coefficient BTU/ft^2, hr, °F
h	Equal to H/k in reciprocal of ft
k	Thermal conductivity BTU/hr, ft^2, °F/ft

$K = k/c_p\rho$ ... Thermal diffusivity ft^2/hr

ρ ... Density #/ft^3

t ... Time hrs

T ... Temperature °F

T_o ... Constant fluid temperature at $t>0$

x ... Length of conduction path ft

$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\mu^2} d\mu$ ... Error function or probability integral

$\operatorname{erfc} x = \left[1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-\mu^2} d\mu\right]$

Stress Equations

σ_r ... Radial stress #/in^2

σ_θ ... Tangential stress #/in^2

σ_z ... Axial stress #/in^2

α ... Linear coefficient of expansion inches /inch

a ... Radius of hole inches

b ... Outer radius of cylinder inches

ν ... Poisson's ratio

r ... Radius inches

T ... Temperature in °F at radius r

TABLE OF CONTENTS

Summary . . . . Page 1

Introduction . . . . Page 2

Generalization of Problem and

Basic Assumptions . . . . Page 6

Numerical Solution for

Temperature Distribution . . . . Page 13

Analytic Solution for Temperature Distribution . Page 19

Determination of Stress Distribution . . . . . . Page 23

Results and Conclusions . . . . Page 27

Recommendations for Further Study . . . . Page 29

Appendix I

Appendix II

Bibliography

SUMMARY

In this paper solutions are presented for the determination of transient thermal stress distributions, in hollow circular cylinders of elastic material, in which the temperature is a function of the radial coordinate as well as time. Indicated solutions are given for an externally insulated cylinder where the surface temperature is a known function of time, and for the case where the temperature-time relations are known for a heating fluid within the cylinder. Numerical results have been included for the specific case of a hollow steel cylinder subjected to the flow of saturated steam. For the particular set of conditions considered the tangential and axial stresses were found to be a maximum at the heated surface at approximately five seconds after the start of heating and attained a magnitude of 52,500 pounds per square inch. Stresses of this magnitude if often repeated could cause fatigue failure and result in rupture of a line carrying any appreciable pressure. It is conceivable that for more extreme conditions the surface stresses might reach magnitudes high enough to cause a permanent set in the material and residual stresses would be present when the cylinder was again cooled to the initial conditions. Stresses of these magnitudes warrant consideration in the design of steam lines that may be subjected to repeated heating cycles.

INTRODUCTION

The thermal stress distribution in hollow circular cylinders caused by a temperature gradient in the radial direction only is well known. The case of steady state heat flow in such cylinders is treated in several textbooks. The problem of stress distribution for the case in which the temperature is a function of both the radial coordinate and time lends itself to solution by the same stress equations, providing that the temperature-time relations are known. Analytic solutions for transient temperature distributions are given by many authors for the case where, from an initial uniform temperature, one or both of the surfaces are subjected to linear temperature gradients, or are suddenly raised to a given value and thus maintained. The process for solution of such problems is to substitute into the already available stress equations, expressions for the temperature as a function of time and radius for the given condition of heat flow. It can be seen then that for the case of the hollow circular cylinder, as for other configurations, the general thermal stress problem is quite straightforward and evolves itself, for the more complex conditions of heat flow, into that of the determination of temperature distributions as a function of time and distance.

Consider now the more general problem of the thermal stress distribution in a cylinder subjected to the flow of a medium

INTRODUCTION

The thermal stress distribution in hollow circular cylinders caused by a temperature gradient in the radial direction only is well known. The case of steady-state heat flow in such cylinders is treated in several textbooks. The problem of stress distribution for the case in which the temperature is a function of both the radial coordinate and time lends itself to solution by the same stress equations, providing that the temperature-time relations are known. Analytic solutions for transient temperature distributions are given by many authors for the case where, from an initial uniform temperature, one or both of the surfaces are subjected to linear temperature gradients, or are suddenly raised to a given value and then maintained. The approach for solution of such problems is to substitute into the already available stress equations, expressions for the temperature as a function of time and radius for the given conditions of heat flow. It can be seen then that for the case of the hollow circular cylinder, as for other configurations, the general thermal stress problem is quite straightforward and resolves itself, for the more complex conditions of heat flow, into that of the determination of temperature distributions as a function of time and distance.

Consider now the more general problem of the thermal stress distribution in a cylinder subjected to the flow of a medium through the cylinder different in temperature to that of the cylinder. In this case the inner and outer surface temperatures will vary with time and in general will not be linear. The solution of such a problem also involves the consideration of a surface heat transfer coefficient which itself may have a magnitude which varies with temperature and consequently with time. Analytic solutions for the transient temperature distribution is a cumbersome double series of Bessel functions which does not readily lend itself to numerical results. However, analytic solutions are available for unidirectional heat flow in the semi-infinite solid in terms of exponentials and the probability function, which are readily adopted to numerical results and can be applied to the hollow circular cylinder within certain limitations. Such solutions are valid for infinitely long thin cylinders and give a close approximation to the temperature distribution for thick cylinders, in which the wall thickness is not large compared to the radius. The usefulness and practicability of such a method to determine transient thermal stresses in hollow cylinders when subjected to a heating medium is the subject of this paper.

In the development of the problem, two separate methods of attack were considered and their results compared. First, a numerical method for the transient temperature distribution based on a difference equation rather than the applicable differential equation

was used to determine the time-temperature gradient. Secondly, a purely analytic method was used to determine the time-temperature relations and in both instances a numerical integrating process was used to evaluate the integrals in the stress equations. It might be noted at this point that the stress equations will give exact results, within the accuracy of the numerical integration, if the time-temperature relations thus determined are the true temperature gradients for the heat flow considered.

It is the intent of this paper to indicate a practical method of solution for such thermal stress problems rather than to obtain accurate numerical results for any specific problem. However, in order to more clearly indicate the feasibility of this approach, a simple numerical problem has been carried through, using average values of thermal and elastic properties with slide-rule accuracy. The solutions of more complex problems are indicated as the method is developed. The problem chosen for its simplicity is that of a hypothetical ten-inch I. D. by twelve-inch O. D. steam line, initially at 100°F suddenly subjected to a saturated steam flow having a temperature of 500°F. It is assumed that the steam starts to condense immediately and remains at the saturation temperature throughout the transient heat transfer process. The surface heat transfer coefficient was taken as an average value over the inner surface of the pipe, considering film type condensation, and assuming

it not effected by temperature. The magnitude of the surface heat transfer coefficient was chosen from average experimental values for film type condensation as found in the literature. The reason for these basic assumptions and their validity are further discussed in the body of the text.

GENERALIZATION OF PROBLEM AND BASIC ASSUMPTIONS

The general differential equation of transient heat flow is obtained from the familiar basic Fourier equation for the conduction of heat ($q = -kA\frac{\partial t}{\partial x}dt$) and a heat balance. Disregarding heat sources or sinks and equating the heat gain to the difference between the heat entering and heat leaving an elemental volume, the general differential equation expressed in rectangular coordinates is:

$$\frac{\partial T}{\partial t} = \frac{1}{c_p\rho}\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)\right]$$

If k, the thermal conductivity, is considered inverable with temperature change and the material is homogeneous and isotropic, k will be the same in all directions and constant.

Then;

$$\frac{\partial T}{\partial t} = \frac{k}{c_p\rho}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

In general, engineering materials can be considered isotropic and homogeneous and this is an ordinary assumption made in engineering calculations. However, the assumption that K is constant may not be valid for all materials within the range of accuracy desired in the resulting calculations. The order of magnitude of

the variation of k with temperature may be seen by citing mild steel as an example, which has a thermal conductivity of approximately 26 BTU/hr ft °F at 32°F and 21 BTU/hr ft °F at 1100°F.

The desired time-temperature-distance relations for the heating or cooling of various shapes are obtained by the integration of the general conduction equation, substituting in the necessary boundary conditions for the case in question. One of the simplest cases is the unidirectional heat conduction in a plate of semi-infinite thickness, with the free surface at $x = 0$. If the term $k/c_p\rho$ is defined as the thermal diffusivity and indicated by the symbol K the general conduction equation for unidirectional heat flow reduces to:

$$\frac{\partial T}{\partial t} = K \left(\frac{\partial^2 T}{\partial x^2}\right)$$

To solve this equation, it is necessary to determine the formulae which will express the initial and boundary conditions which the temperature satisfies. If a semi-infinite solid with its plane face at $x = 0$, initially at the constant temperature of zero and suddenly subjected to a heating medium, which has allied with it a surface heat transfer coefficient H, the initial and boundary conditions are:

Initial condition $T = 0$ WHEN $t = 0$

Boundary condition $-\frac{\partial T}{\partial x} + \frac{H}{k}(T_s - T_a) = 0$ AT $x = 0$

or defining H/k as h the boundary condition is:

$$-\frac{\partial T}{\partial x} + h(T_s - T_a) = 0 \quad \text{AT } x = 0$$

The solution of the differential equation for unidirectional heat conduction with these initial and boundary conditions will give the temperature as a function of time and distance for the semi-infinite solid. Such a solution will also be valid for a plate of finite thickness having a large ratio of surface to thickness, if it is heated, as before, at the face x = o and the other face is perfectly insulated. To use such a solution for a hollow circular cylinder, externally insulated, and subjected to the flow of a heating medium it must be assumed that the curved section of the cylinder can under certain circumstances be considered as a flat insulated plate. This assumption will necessarily introduce an error in the transient temperature distribution thus determined for the cylinder and will be a function of the ratio of thickness to radius. This fact must be considered in the calculation of stress distributions for stresses thus determined are only as reliable as the temperature distributions used in their calculation.

To verify the validity of this assumption the applicable difference equations may be set up for the plane plate and the hollow circular cylinder and the results compared. As explained under the heading "Numerical Solution of Temperature Distribution",

the numerical method converges rapidly to the analytic solution. This being the case, a comparison of numerical methods should also justify the use of the analytic solution for the plane semi-infinite solid for stress calculations for the hollow circular cylinders within the same range of accuracy.

Taking a section of the plane solid of unit width, Δx thick, and dividing it up as shown in the figure below, a heat balance gives:

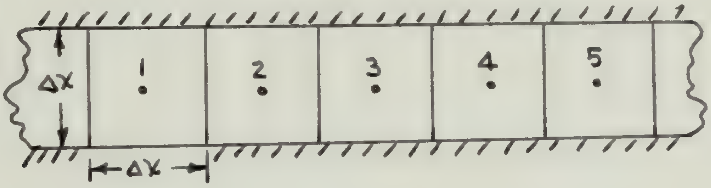


FIGURE A

$$k(T_1-T_2)\Delta t + k(T_3-T_2)\Delta t = C_p\rho\Delta x^2 (T_2'-T_2)$$

WHERE: $T_2' = T_2$ AT $(t+\Delta t)$

OR $$(T_1-T_2)+(T_3-T_2) = M(T_2'-T_2)$$

WHERE: $M = \frac{C_p\rho\Delta x^2}{k\Delta t}$

THEN: $$T_2' = T_2 + \frac{T_1-2T_2+T_3}{M} \qquad ①$$

$$= T_2 + \frac{T_1-2T_2+T_3}{4}$$ WHERE: $M=4$

A similar heat balance for the circular section where the area between sections progressively increases gives:

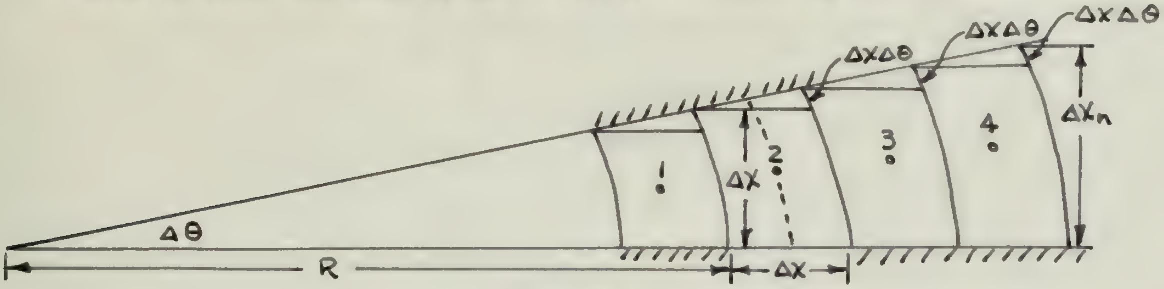


FIGURE B

$$k(T_1-T_2)\Delta t + k(1.0+\Delta x\Delta\theta)(T_3-T_2)\Delta t = C_p\rho\Delta x^2\left(1+\frac{\Delta x\Delta\theta}{2}\right)(T_{2r}'-T_2)$$

WHERE: $T_{2r}' = T_2$ AFTER $(t+\Delta t)$ IN CYLINDER

OR $$(T_1-T_2)+(1.0+\Delta x\Delta\theta)(T_3-T_2) = M\left(1+\frac{\Delta x\Delta\theta}{2}\right)(T_{2r}'-T_2)$$

SOLVING FOR T_{2r}' : LETTING $M=4$

$$T_{2r}' = T_2 + \left[\frac{T_1 - T_2(2+\Delta x\Delta\theta) + T_3(1+\Delta x\Delta\theta)}{M\left(1+\frac{\Delta x\Delta\theta}{2}\right)}\right] \qquad \textcircled{2}$$

It can be seen that equations 1 and 2 are identical in form, the only difference being the addition of the term $\Delta x\Delta\theta$ for the circular cylinder. For the ordinary problem, in order to satisfy the basic assumptions of such a numerical solution, Δx must be chosen as a low order term. This means that the radius of the cylinder will be large as compared to Δx and thus $\Delta\theta$ must also

be a low order term. The magnitude of these values may readily be seen by taking as an example a hollow cylinder of five inch inside radius and a $\Delta x = .01$ ft (Note that this is the largest value of Δx chosen for numerical calculation) then:

$$\Delta\theta \cong \tan\Delta\theta = \frac{.01}{5} \times 12 = .024 \text{ RAD.}$$

and $\Delta x \Delta\theta = .00024$ (Which is essentially a second order term).

Then for the use of a modulus $M = 4$:

$$T'_{2r} = T_2 + \left[\frac{T_1 - (2.00024)T_2 + (1.00024)T_3}{4.00048}\right] \qquad ③$$

A comparison of equations 3 and 1 shows that this point, namely at a radius of five inches, T'_2 and T'_{2r} are practically identical. However, in the numerical solution for temperature distribution, the temperature at any point is used to determine the temperature at the next adjacent point, etc., through the solid. This means that the error in using equation 1 instead of equation 2 will be an accumulative error and places a restriction upon the thickness of the cylinder for a given error. This fact perhaps can be seen more clearly by noting from Figure A that;

$$\Delta x_n = \Delta x + n\Delta x\Delta\theta$$

Where: n is the number of steps to be taken for the given thickness chosen.

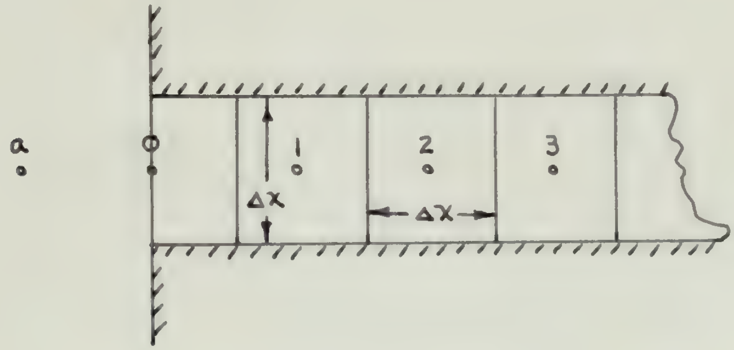
It should also be noted that the error is not a function of Δx alone but a function of the product of Δx and $\Delta\theta$, which essentially means that for a given radius cylinder and wall thickness the error involved will be the same regardless of the choice of Δx. With these facts in mind, it is apparent that the limiting factor is the ratio of wall thickness to radius regardless of their absolute magnitudes.

Another pertinent observation is that at the first instant of heating the two equations almost coincide and the error in temperature distributions increases with each step. As the temperature distributions thus determined, only enter the stress equations in the terms $\int_a^b Tr\,dr$ and $\int_a^r Tr\,dr$ the true criterion for the error involved in stress calculations would be to plot temperature times radius vs. radius curves for both methods and compare their areas.

It is the author's opinion that the error involved due to this basic assumption will be of the same order of magnitude or less than the errors inherently involved in thermal and elastic properties, where ratios of wall thickness to radius found in ordinary steam lines are to be used.

NUMERICAL SOLUTION FOR TEMPERATURE DISTRIBUTION

Unidirectional heat flow will occur in a flat slab at a distance from the edges and also in an insulated rod of uniform section. As far as temperature distribution is concerned, the size and shape of the rod is immaterial and is only of consequence in calculating total heat flow. Consider such a rod of unit width, a thickness Δx and divided up into sections Δx long as indicated in the figure below:



If now a finite surface heat transfer coefficient H is considered between the surface (0) and the ambient fluid temperature (a) then for the point (0) a heat balance gives:

$$H \Delta x (T_a - T_0) \Delta t = - k (T_1 - T_0) \Delta t + C_p \rho \Delta x^2 (T_0' - T_0)$$

Where $T_0' = T_0 \text{ AT } (t + \Delta t)$

The finite difference equation thus obtained is based upon the following assumptions:

a. That Δt be taken small enough so that there is a negligible error in heat flow calculated by the term $(T_1-T_2)/\Delta x$.

b. That Δt be taken small enough so that there is a negligible error in neglecting any but adjacent areas (Δx by Δx).

c. That Δx be taken small enough so that the temperature taken at the center of the element can be used to compute the heat capacity.

Define:

$$M = \frac{C_p \rho \Delta x^2}{k \Delta t} = \frac{\Delta x^2}{K \Delta t}$$

$$N = \frac{H \Delta x}{k}$$

Using these relations:

$$N(T_a - T_0) + (T_1 - T_0) = \frac{M}{2}(T_0' - T_0)$$

Solving for T_0'

$$T_0' = \frac{2N}{M} T_a + \left(1 - \frac{2N+2}{M}\right) T_0 + \frac{2}{M} T_1$$

This equation may be written in terms of "influence factors" as follows:

$$T_0' = F_{a0} T_a + F_{00} T_0 + F_{01} T_1$$

Where:

$$F_{a0} = \frac{2N}{M}, \quad F_{00} = \left(1 - \frac{2N+2}{M}\right), \quad F_{01} = \frac{2}{M}$$

The subscripts indicate in order, the influence that the present temperature has on the point whose temperature at $(t + \Delta t)$ is to be found for the period of time Δt. In other words, if the new temperature at point (0) is to be found after the time Δt, F_{ao} indicates the influence of point (a) on the temperature at point (0) during the time interval Δt.

If T_o^1 is to depend upon T_o in a positive sense, then:

$$F_{oo} = 1 - \frac{2N+2}{M} \geqq 0$$

$$\text{OR} \quad M - 2N + 2 \geqq 0$$

$$M \geqq 2N+2$$

Since all of the quantities entering into N are inherently positive, it is apparent that M must be larger than two , for the case where a surface transfer coefficient is involved. Depending upon the magnitude of the surface transfer coefficient and the thermal constants of the material, this may place a limitation upon a convenient averaging process. The value of M=4 was found suitable for numerical temperature calculations for steel.

The general method is to use the weighting process, as described above, for surface temperature calculations and the simpler averaging process for interior temperature determinations. It is convenient to choose a scale such that fractions can be neglected.

If the fraction 1/2 was consistently added or dropped, an error would tend to accumulate. Therefore, a rule of dropping the 1/2 after even digits and adding 1/2 after odd digits can be adopted, as; 1/2-0, 3/2-2, 5/2-2, 7/2-4, etc.

The choice of the modulus M is discussed in reference (5). The larger the modulus M the sooner the numerical results will converge to the analytic solution (i.e. in less steps) and the greater the number of calculations necessary to cover the same time period. After due consideration of the work involved, a modulus M=4 was chosen to obtain rapid convergence to the analytic solution during the early time intervals.

Now consider the heating of a one-inch thick plate insulated on one surface and subjected to a heating medium on the other surface. For simplified calculation, consider K and H as constant. By choosing several values of Δx (and the compatible Δt) for a modulus M=4, surface temperature vs. time curves can be plotted to determine the validity of the choice of the magnitude of Δx. As the accuracy of the solution is based on the assumption that Δx be small, such a plot should indicate any variation in the results, due to an increase in the magnitude of Δx.

For ordinary Steel:

$K \cong .50\ ft^2/hr$

$k \cong 25$ BTU/hr, ft, °F

Assume: $H = 1000$ BTU/hr, ft^2, °F (A fair approximation for film-type condensing steam — see Appendix I)

Sample determination of N, F_{ao}, F_{oo}, F_{oi} and Δt.

Let $\Delta x = .01$ FT. $\Delta x = 12 \times .01 = .12$ INCHES

Then: $$N = \frac{H}{k} \Delta x = \frac{1000 \times .01}{25} = .40$$

$$M = \frac{\Delta x^2}{K \Delta t} \quad \text{OR} \quad \Delta t = \frac{\Delta x^2}{4K} \quad \text{WHERE } M = 4$$

$$\Delta t = \frac{(.01)^2}{4 \times .5} = 0.5 \times 10^{-4} \text{ HRS.}$$

$$\Delta t = 0.5 \times 10^{-4} \times 3600 = .18 \text{ SEC.}$$

$$F_{ao} = \frac{2N}{M} = \frac{2 \times 0.4}{4} = 0.2$$

$$F_{oo} = \left(1 - \frac{2N + 2}{M}\right) = \left(1 - \frac{0.8 + 2.0}{4}\right) = 0.3$$

$$F_{io} = \frac{2}{M} = \frac{2}{4} = 0.5$$

Similarly for the other values of Δx:

TRIAL	Δx IN FT	Δx IN INCHES	Δt HRS	Δt SEC	N	F_{ao}	F_{oo}	F_{io}
1	0.01	.12	$.50 \times 10^{-4}$	.18	0.4	0.20	0.30	0.5
2	0.0075	.09	$.281 \times 10^{-4}$	.101	0.3	0.15	0.35	0.5
3	0.0050	.06	$.125 \times 10^{-4}$	.045	0.2	0.10	0.40	0.5
4	0.0025	.03	$.3125 \times 10^{-4}$	.01125	0.1	0.05	0.45	0.5

The equations for the surface temperature become:

(1) $$T_0' = 0.2 T_a + 0.30 T_0 + 0.5 T_1$$

(2) $$T_0' = 0.15 T_a + 0.35 T_0 + 0.5 T_1$$

(3) $$T_0' = 0.10 T_a + 0.40 T_0 + 0.5 T_1$$

(4) $$T_0' = 0.05 T_a + 0.45 T_0 + 0.5 T_1$$

One set of calculations will be given at this point to indicate the form and procedure. The results only for other numerical calculations are included in Appendix II and plotted on the enclosed curve sheets.

$\Sigma \Delta t$ Sec	a	0	1	2	3	4
0	500	100	100	100	100	100
.18	500	1800 180	100			
.36	500	2040 204	480 120	100		
.54	500	2212 221	544 136	420 105	100	
.72	500	2343 234	598 150	446 112	405 101	100
.90	500	2452 245	646 162	475 119	414 104	401 100

ANALYTIC SOLUTION FOR TEMPERATURE DISTRIBUTION

To determine the time-temperature-distance relations in the semi-infinite solid for the case where the initial temperature is zero and the surface temperature is a function of time $\phi(t)$, the following equation is valid:

$$T = \frac{2}{\sqrt{\pi}} \int_{x/2\sqrt{Kt}}^{\infty} \phi\left(t - \frac{x^2}{4K\mu^2}\right) e^{-\mu^2} \, d\mu \qquad ①$$

Also for the case of the semi-infinite solid, initially at zero temperature, which is heated at the surface $x=0$ by radiation from a medium at constant temperature T_a, the applicable equation is:

$$\frac{T}{T_a} = \operatorname{erfc} \frac{x}{2\sqrt{Kt}} - e^{hx + h^2Kt} \operatorname{erfc} \left\{ \frac{x}{2\sqrt{Kt}} + h\sqrt{Kt} \right\} \qquad ②$$

Where erfc is defined as one minus the probability function or error integral.

$$\operatorname{erfc} x = \left(1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-\mu^2} d\mu\right)$$

It can readily be shown by direct substitution into the differential equation of unidirectional heat flow that these equations do satisfy the differential equation and initial and boundary conditions.

Equation (1) was given to emphasize the fact that whenever the surface temperature of the semi-infinite solid is known as a function of time $\phi(t)$, and this function is continuous and can be integrated over the time interval (in equation (1)) to be considered, the resulting time-temperature-distance relations for the solid are then determined. This opens the avenue for the solution of a number of interesting problems. As noted before, a numerical solution for surface temperature is readily adopted to thermal properties of the material which vary with temperature, as well as a surface heat transfer that is a function of temperature. This method can also be extended to the case where the temperature of the heating medium is some known function of time. However, it should be noted that such a solution for surface temperature, under the above stated conditions may become very tedious, as the influence coefficients may have to be calculated for each step of the process as well as choosing a Δt that is small. Although the numerical solution itself, if carried through the solid to the desired depth, will give the time-temperature-distance relations directly, the desired surface temperature $\phi(t)$ may be determined over a shorter interval with the resulting saving of much numerical work.

Figures (C) and (D) show the determination of the surface temperature relations $\phi(t)$ for a short interval of time. Figure (D) shows that the surface temperature vs. time curves very closely approximate a straight line on a log-log plot, and can readily be put into an analytic expression.

Equation (2) has been used to determine the time-temperature-distance relations in a perfectly-insulated, one-inch-thick plane solid of large surface area. The results of this can be used as a close approximation of the true transient temperature gradient in an insulated 10-inch inside diameter hollow cylinder of one-inch wall thickness.

For the ultimate calculation of the stress distribution, it is convenient to plot temperature-distance relations for various intervals of time, in order to perform a graphic or numerical integration of the integrals in the stress equation. Figure (B) shows a plot of these relations and shows how closely the numerical method parallels the analytic solution. Figure (E) gives the equivalent temperature times distance vs. distance relations, and indicates a graphic integrating process. It might be preferable and possibly a little more accurate to use a numerical integrating scheme such as the trapezoidal or Simpson's rule.

Numerical results are not as tedious as a first inspection of equation (2) might indicate. Tabulated values of the error integral

are available and many of the parameters are constant for one instant of time. A simple tabulation of the quantities given below will materially facilitate calculation, all other parameters being constant for a given instant of time.

(1) x IN FT

(2) $\frac{x}{\sqrt{Kt}}$

(3) $\text{erfc}\ \frac{x}{\sqrt{Kt}}$

(4) $\text{erfc}\ \left(\frac{x}{\sqrt{Kt}} + h\sqrt{Kt}\right)$

(5) hx

(6) $hx + h^2Kt$

(7) $e^{hx + h^2Kt}$

(8) T

The results of calculations made for the semi-infinite solid for a penetration of one inch are given in appendix to correlate the data used to plot curves.

DETERMINATION OF STRESS DISTRIBUTION

If the transient temperature distributions determined by either numerical or analytic methods for the semi-infinite solid under the conditions specified are considered good approximations of true temperature distributions in the hollow cylinder, calculation of stress distributions are straight forward. At any instant of time the stress becomes a function of the thermal and elastic properties of the material for the geometric configuration considered, and can be readily calculated. The following thermal stress equations for the finite hollow cylinder are given by Timoshenko:

$$\sigma_r = \frac{\alpha E}{(1-\gamma)} \frac{1}{r^2} \left\{ \frac{r^2-a^2}{b^2-a^2} \int_a^b Tr\,dr - \int_a^r Tr\,dr \right\}$$

$$\sigma_\theta = \frac{\alpha E}{(1-\gamma)} \frac{1}{r^2} \left\{ \frac{r^2+a^2}{b^2-a^2} \int_a^b Tr\,dr - \int_a^r Tr\,dr - Tr^2 \right\}$$

$$\sigma_z = \frac{\alpha E}{(1-\gamma)} \frac{1}{r^2} \left\{ \frac{r^2+a^2}{b^2-a^2} \int_a^b Tr\,dr - \int_a^r Tr\,dr \right\}$$

These equations are based on thermal and elastic properties that do not vary with temperature. Within this limitation they are exact for an infinitely long cylinder of any wall thickness and any given temperature distribution so long as it is in the radial direction only. The principal planes of stress are in the radial,

axial, and tangential directions with normal stress acting on each and stress acting along a plane perpendicular to this axis are independent of the position of this plane.

In order that the solution be exact when a cylinder of finite length be chosen, boundary conditions require that external forces equal in value and distribution to the stresses in the axial direction act on the ends. In the usual case such forces will not be present, so that the stresses near the ends will be very different than those calculated by this method. This effect diminishes as the distance from the ends is increased and practically disappears at a point sufficiently far away. Writers differ as to the distance from the ends where this point is located and it actually depends upon the ratio of the thickness to cylinder diameter. For the purpose of our problem, this influence will be considered negligible.

It is interesting to note that in the case where the surface of a solid, initially at uniform temperature, is suddenly raised to a given temperature and thus maintained, the maximum stress occurs at the surface at that instant. In fact, the maximum stress (i.e. tangential and axial stresses which are equal at the surface) will always occur at the heated surface for a solid initially at uniform temperature. The stresses thus produced are equivalent to the stress produced in a thin plate for the same temperature rise in which a change of area is prevented. This stress is only momentary and

rapidly drops as the increase of temperature spreads inward and allows an expansion of the outer layer. Most published solutions have this common boundary condition, namely, an instantaneous surface heat source.

For the case of radiation into a solid from a fluid, where the surface temperature is a function of time and equal to the initial solid temperature at the first instant of heating, the conditions are vastly different. The maximum surface stresses will not occur at the first instant of heating but at some later time, and depending upon the conditions of heat flow may even occur at the steady state condition. For the case of the sudden rise of the surface temperature of a solid, initially at constant temperature and ultimately coming to a steady state condition, it is essentially going from an unstressed condition to a stressed condition. In the problem chosen, however, the cylinder at an initial uniform temperature with the outer surface perfectly insulated will, upon heating of the inner surface, be taken from one unstressed condition to another as the cylinder finally reaches the temperature of the heating medium. It is reasonable to assume then that maximum stress conditions will occur at some intermediate time.

Calculated values of inner surface stress are tabulated in Appendix II for a one-inch thick, ten-inch I. D. circular steel

cylinder along with the stress distribution across its section at the time of maximum stress. The cylinder being insulated on its outer surface, initially at 100°F and suddenly subjected to a saturated steam flow which is maintained at 500°F. The results of these calculations are graphically represented by figures (E), (F) and (G) and will be further discussed in the results and conclusions.

APPENDIX I

Accurate thermal stress determinations in steam lines are usually linked with a knowledge of the surface heat transfer coefficient. The matter of the physical phenomena involved as well as the magnitude of such coefficients is the subject of a good deal of conjecture. This coefficient is defined by the heat flow equation:

$$q = HA(T_s - T_a)$$

Where: q - Heat Flow

H - Surface transfer coefficient

A - Transfer area

T_a- Fluid temperature

T_s- Solid surface temperature

Values of H determined by experimental means are in general more reliable than those determined analytically, and particularly for condensing steam.

There are two types of condensation, namely, dropwise and film condensation. Either or a combination of the two types may exist, depending largely upon the cleanliness of the fluid, cleanliness of the condensing surface and the surface conditions of the material concerned in the condensation process. In any case, heat transfer coefficients for the dropwise process are in general many times higher than for film condensation, but are more difficult to determine.

Authors differ widely as to the magnitude of the difference between the two types, giving coefficients from 4 to 20 or more times larger for the more efficient dropwise process. It has been determined that in general clean steam with or without non-condensible gases always condenses in a film on clean surfaces, whether rough or smooth.

Experimentally determined values of heat transfer coefficients are given in terms of an average value over a prescribed area, discounting the fact that the value of the coefficient may vary considerably over the area depending upon the film thickness. Nusselt in an analytic investigation of heat transfer coefficients on cylindrical surfaces found the film thickness and consequently the transfer coefficient to vary around a horizontal tube. His analysis shows the minimum film thickness and consequent maximum transfer coefficient to exist a few degrees on either side of the top of a horizontal tube. His investigation is equally valid for condensation inside or outside of a tube. By integration around the tube an average coefficient was determined. Experimental determinations have shown the average coefficient thus determined to be somewhat in error and correction factor curves are available for use with Nusselt's equation to give representative values of the surface heat transfer coefficient for condensing steam on tubes.

If now dropwise condensation were to be considered, the stress problem immediately becomes very involved for the local transfer coefficient would vary appreciably from droplet to unwetted surface and the heat flow would no longer be unidirectional. For such a problem an entirely different approach would have to be considered. The value of H-1000 BTU/hr ft^2 °F used in the numerical calculations is an arbitrary value chosen from the references for film-type condensation of steam on a horizontal tube.

RESULTS AND CONCLUSIONS

It is apparent from the magnitude of the inner surface stress determined, for the numerical problem considered, that thermal stresses under severe conditions of repeated heating cycles could contribute to ultimate rupture of a line under pressure. The tangential stress on the inner surface due to pressure in the line will be tensile and to some degree counteract thermal stresses. However, for the 10-inch I. D. by 12-inch O. D. line considered, the stress due to a pressure of 515 $\#/\text{in}^2$ in the line would be approximately $\sigma_r = 500\ \#/\text{in}^2$ in compression and $\sigma_\theta = 1{,}640\ \#/\text{in}^2$ in tension. These stresses are extremely small in magnitude as compared to the thermal stresses determined.

It should be remembered that this paper was motivated by a desire to formulate a practical approach to the actual thermal stress problems in hollow circular cylinders which are extremely difficult if not impossible by purely analytic approach. Necessarily, many assumptions were made, and to some degree they have been justified in the text of the paper. It is obvious then that the results of the numerical calculations should be substantiated by an experimental investigation before they are accepted as accurate enough for practical use.

As little data is available on the magnitude or variation of surface transfer coefficients, for condensing steam in hollow

circular cylinders under high temperatures and pressures, the value chosen could be considerably in error, as it is a value extrapolated from experimental results for condenser tubes under lower temperatures and pressures. It is conceivable that a consideration of the change of the magnitude of the surface transfer coefficient with temperature difference across the film as well as the change in thermal properties with solid temperature would materially affect the thermal stresses determined. The simplifying assumption that the line was perfectly insulated would also constitute some error in stress calculations beyond the time at which the outer surface temperature begins to rise. Although under steady state conditions insulations are available that are extremely effective, under transient conditions, temperature distributions will necessarily be affected by the heat capacity of the insulation. This condition was not considered in the numerical problem which injects some error in the resulting stress determinations for times beyond approximately five seconds. Perhaps, with some of the qualifying assumptions in mind, this paper will stimulate further investigations along the same lines toward a usable practical solution for thermal stresses in steam lines.

slender cylinders under high temperatures and pressures, the value chosen could be considerably in error, as it is a value extrapolated from experimental results for pressure tubes under lower temperatures and pressures. It is conceivable that a consideration of the change of the magnitude of the surface transfer coefficient with temperature differences across the film as well as the change in thermal properties with solid temperature would materially affect the thermal stresses determined. The simplifying assumption that the film was perfectly insulated would also contribute some error in stress calculations beyond the time at which the outer surface temperature begins to rise. Although under steady state conditions insulators are available that are extremely effective, under transient conditions, temperature distributions will necessarily be affected by the heat capacity of the insulation. This condition was not considered in the numerical problem which imparts some error to the resulting stress determinations for times beyond approximately five seconds. Perhaps, with some of the qualifying assumptions in mind, this paper will stimulate further investigations along the same lines toward a usable practical solution for thermal stresses in these films.

RECOMMENDATIONS FOR FURTHER STUDY

Further study of the practical problem of thermal stresses in hollow circular cylinders during actual transient boundary conditions falls into the several broad categories listed below:

(a) The experimental determination of the true boundary conditions that may exist in the actual problem during the transient state, such as those conditions that may exist when a steam line is suddenly subjected to a flow of high temperature steam.

(b) An experimental verification of the numerical example given in the text as to the time at which maximum stress occurs, its magnitude, as well as the stress distribution across the section at time of maximum stress.

(c) An experimental study of the magnitude of the surface transfer coefficient under high temperatures and pressures in hollow circular cylinders. How the value of such a coefficient varies with pressure, temperature difference across the condensate film, and angular position in a horizontal cylinder.

(d) A further justification of the use of temperature distributions as determined for unidirectional heat flow in the stress calculations for hollow circular cylinders.

There are two immediate avenues of approach to the last category, first, by direct comparison of the temperature distributions as determined by purely numerical methods for the plane plate and hollow circular cylinder, and, secondly, such a temperature comparison could be made by the use of a conformal mapping process from a plane finite section to a circular section. A plot could then be made of the error in stress calculations as a function of the ratio of wall thickness to radius, which would verify the usefulness of the approximate method for a problem of given geometry.

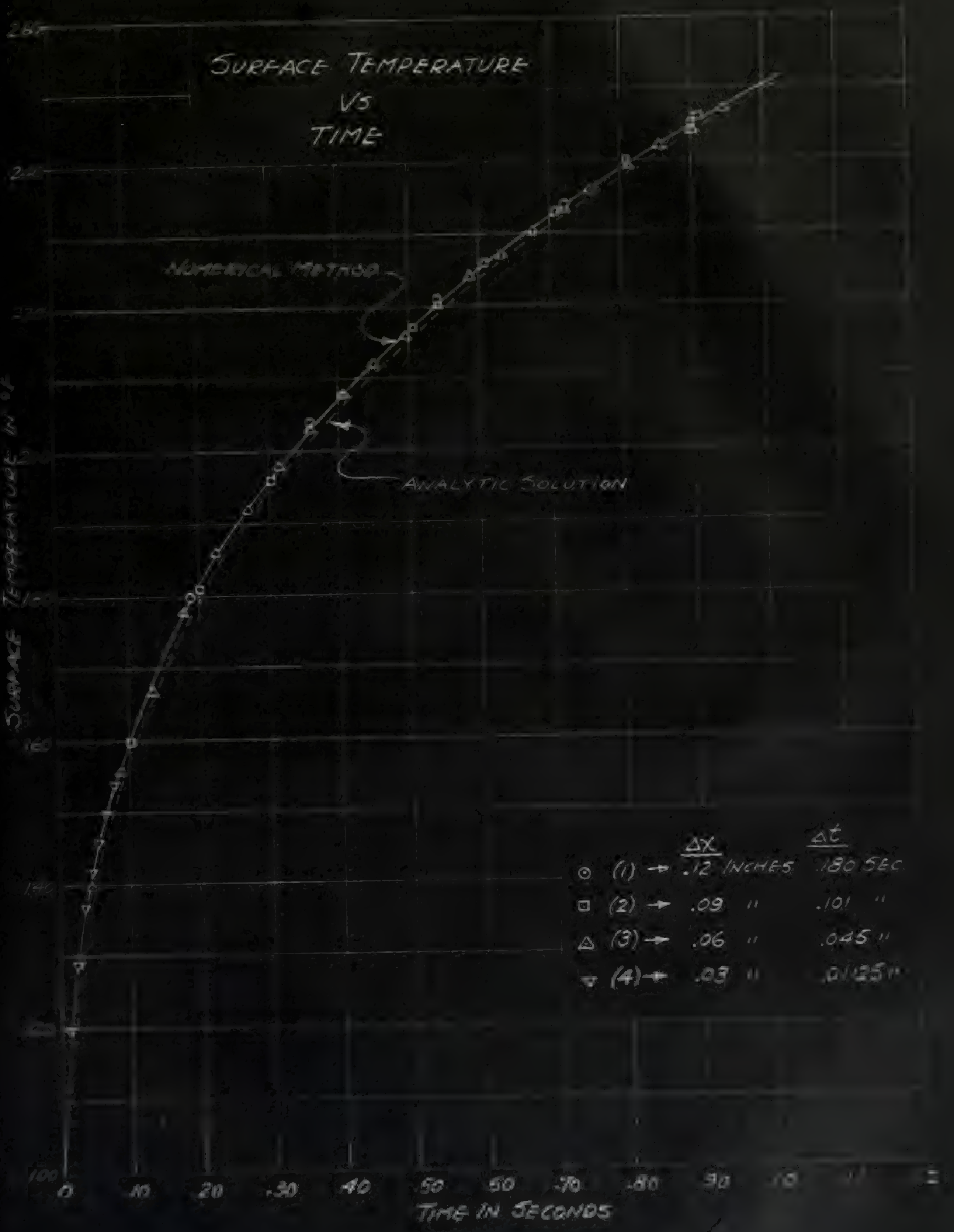


FIGURE A SEMI-INFINITE SOLID

INITIAL TEMP. - 100°F HEATING MEDIUM - 500°F H=1000

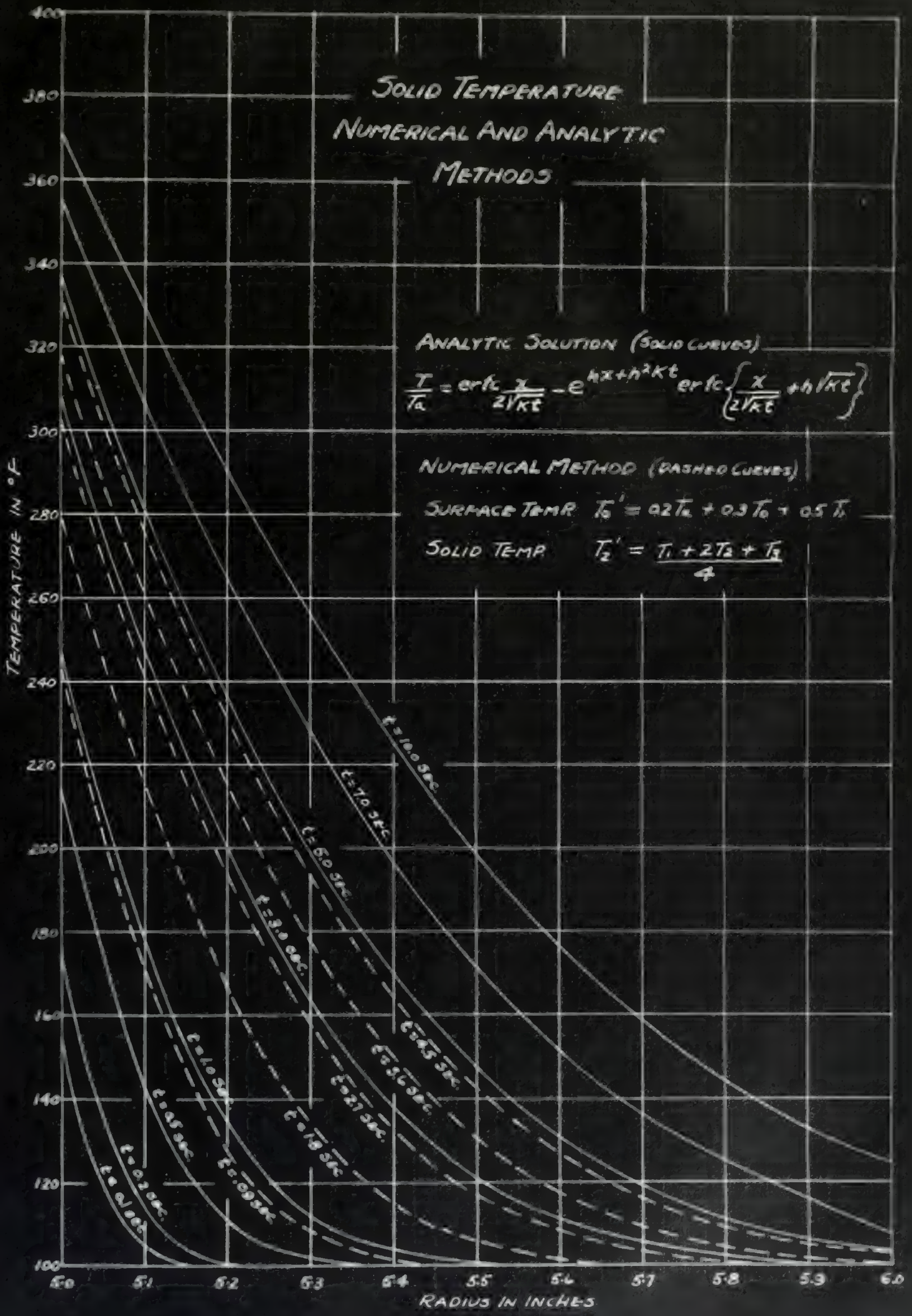


FIGURE B

INITIAL TEMP. - 100°F HEATING MEDIUM - 500°F H=1000

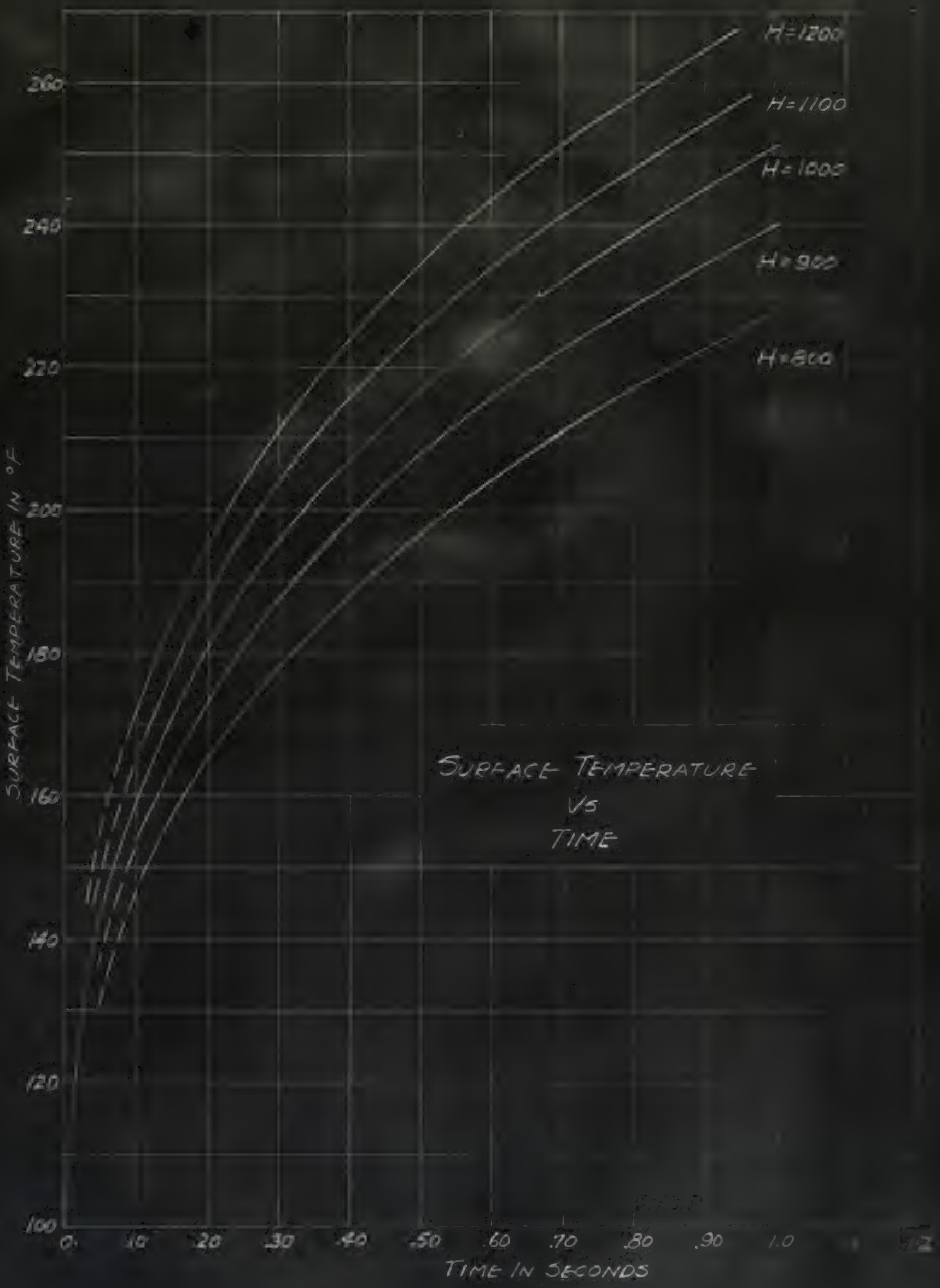


FIGURE C SEMI-INFINITE SOLID

INITIAL TEMP. - 100°F HEATING MEDIUM - 500°F

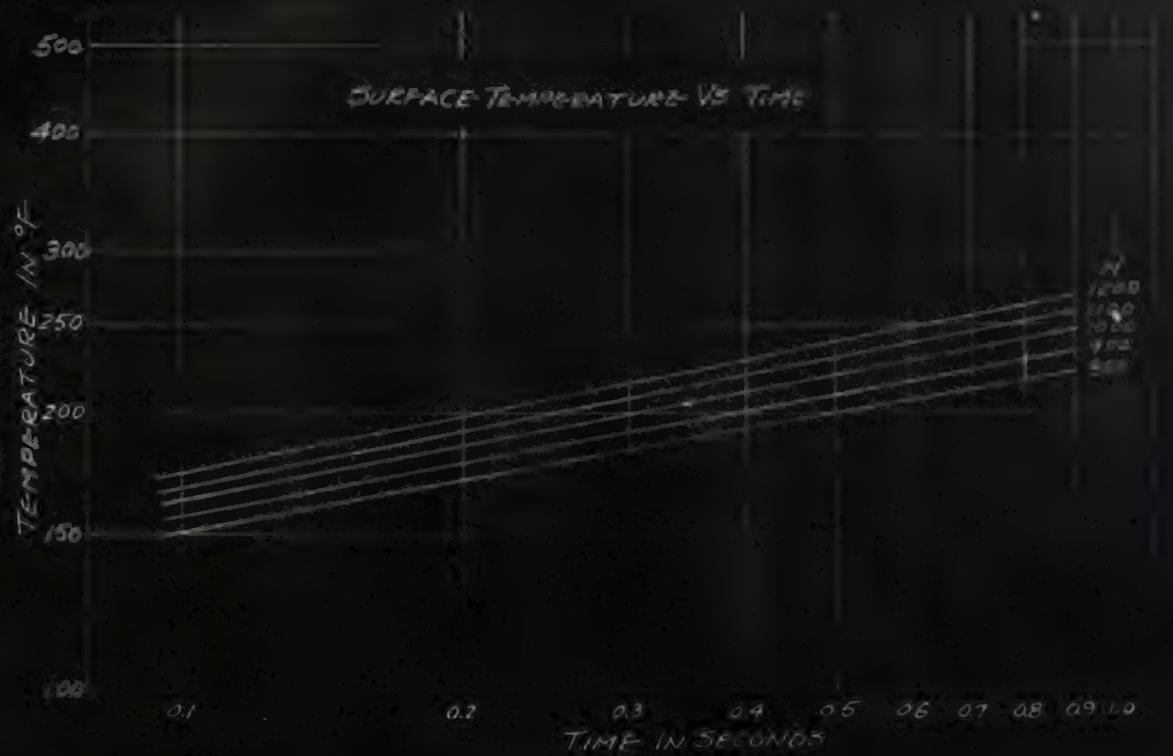


FIGURE D SEMI-INFINITE SOLID

INITIAL TEMP. -100°F HEATING MEDIUM - 500°F

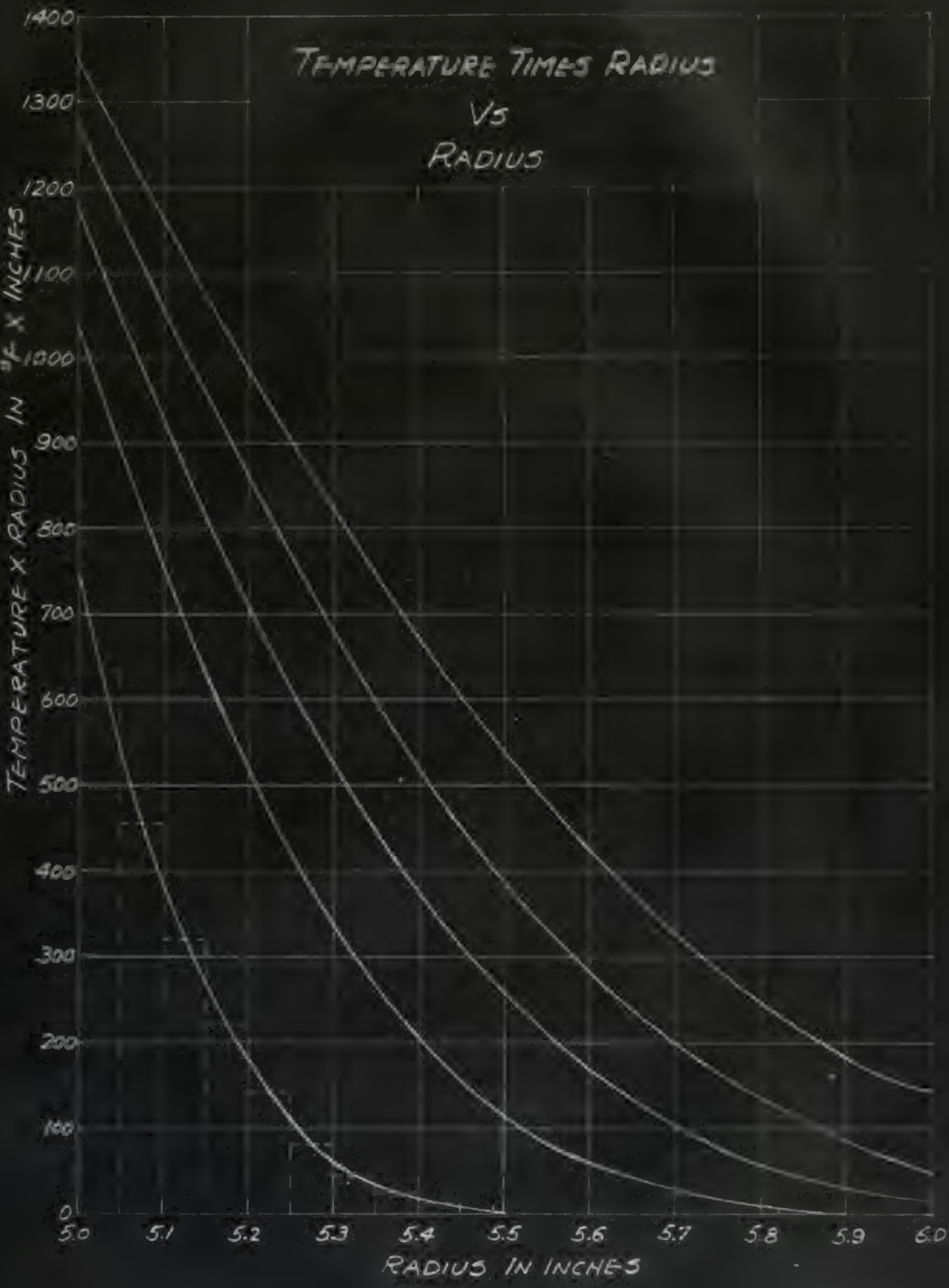
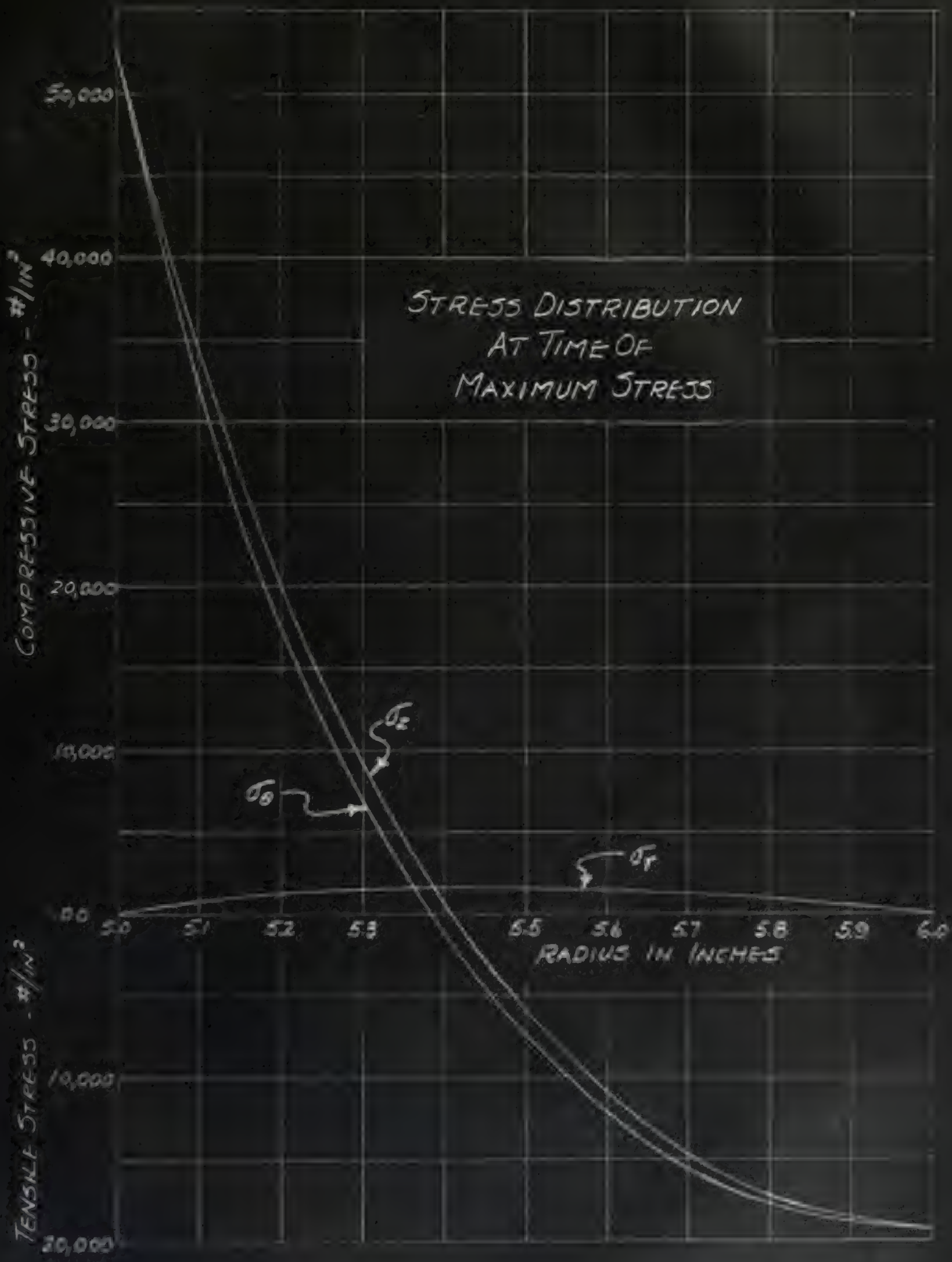


Figure E 10" ID by 12" OD Cylinder - Initially At 100°F Suddenly Subjected To The Flow Of 500°F Saturated Steam.

FIGURE F 10" ID BY 12" OD CYLINDER - INITIALLY AT 100°F SUDDENLY SUBJECTED TO THE FLOW OF 500°F SATURATED STEAM.



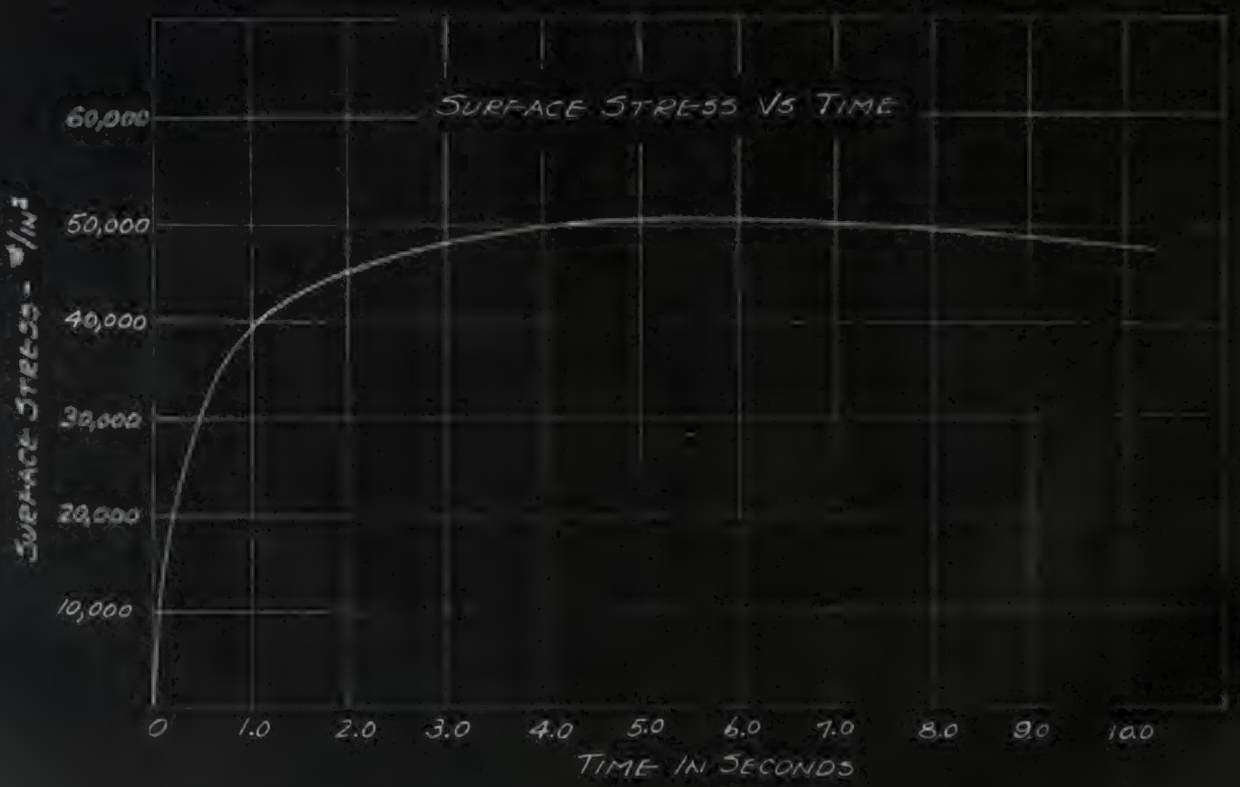


FIGURE G 10" ID BY 12" OD CYLINDER - INITIALLY AT 100°F SUDDENLY SUBJECTED TO THE FLOW OF 500°F SATURATED STEAM.

APPENDIX II

Tabulated values used to plot curves

Figure (A) - (Numerical method - Surface temp.)

$$T_o' = \frac{N}{2}T_a + \left(\frac{1}{4} - \frac{N+1}{2}\right)T_o + \frac{1}{2}T_1 \quad \text{(NOTE: } M=4\text{)}$$

$$T_o = 100, \quad T_a = 500, \quad H = 1000, \quad K = .50$$

Δx = .12 inches		Δx = .09 inches		Δx = .06 inches		Δx = .03 inches	
Δt = .180 sec		Δt = .101 sec		Δt = .045 sec		Δt = .01125 sec	
Tin°F	t in Sec	Tin°F	t in Sec	Tin°F	t in Sec	Tin°F	t in Sec
100	0	100	0	100	0	100	0
180	.18	160	.101	140	.045	120	0.01125
204	.36	181	.202	156	.090	129	0.02250
221	.45	196	.303	167	.135	137	0.03375
234	.72	208	.404	178	.180	142	0.045
245	.90	217	.505	186	.225	146	0.05625
		226	.606	192	.270	150	0.06750
		233	.707	198	.315	154	0.07875
		240	.808	203	.360		
		246	.909	208	.405		
				212	.450		
				216	.495		
				220	.540		
				224	.585		
				227	.630		
				230	.675		
				233	.720		
				236	.765		
				239	.810		
				242	.855		
				244	.900		
				247	.945		

Figure (A) - (Analytic method - Surface temp.)

$$\frac{T}{T_a} = 1.0 - e^{h^2Kt}\,\mathrm{erfc}\left(h\sqrt{Kt}\right)$$

$T_o = 100°F$, $T_a = 500°F$, $h = 40$, $K = .50$

T in °F	t in Sec
143.0	0.05
159.6	0.1
177.6	0.2
192.4	0.3
204.3	0.4
215.2	0.5
221.7	0.6
229.7	0.7
236.4	0.8
242.0	0.9
247.0	1.0

Figure (B) - (Numerical method - Solid temp.)

Surface temp. — $T_o' = 0.2 T_a + 0.3 T_o + 0.5 T_1$

Solid temp. — $T_2' = \frac{T_1 + 2T_2 + T_3}{4}$

$T_o = 100°F$, $T_a = 500°F$, $H = 1000$, $K = .50$

Radius in inches	t=0.9 Sec T - °F	t=1.8 Sec T - °F	t=2.7 Sec T - °F	t=3.6 Sec T - °F	t=4.5 Sec T - °F
0	245	281	303	318	331
.12	161	203	231	251	267
.24	117	151	177	198	215
.36	103	121	141	159	174
.48	100	107	119	133	145
.60		102	108	117	127
.72		100	103	107	114
.84			101	103	109
.96			100	101	103
1.08				100	101
1.20					100

Figure (B) - Analytic Method - Solid Temp.

$T_o = 100^{o}F$, $T_s = 500^{o}F$, $H = 1000$, $k = 25$, $h = 40$, $K = 50$

t = 0.1 Sec		t = 0.2 Sec		t = 0.5 Sec		t = 1.0 Sec	
T - °F	Radius in "	T - °F	Radius in "	T - °F	Radius in "	T - °F	Radius in "
158	5.0	174.8	5.00	214.4	5.00	247.5	5.00
138.9	5.02	146.2	5.03	188.8	5.03	208.7	5.05
124.6	5.04	130.7	5.06	166.2	5.06	176.8	5.10
114.1	5.06	116.4	5.09	148.0	5.09	152.0	5.15
107.6	5.08	108.7	5.12	132.5	5.12	134.0	5.20
103.8	5.10	104.0	5.15	122.9	5.15	120.5	5.25
101.8	5.12	101.7	5.18	115.2	5.18	112.1	5.30
100.8	5.14	100.7	5.21	109.6	5.21	106.8	5.35
100.26	5.16	100.3	5.24	105.8	5.24	103.6	5.40
				103.5	5.27	101.7	5.45
				101.9	5.30	101.1	5.50
				101.0	5.33		
				100.5	5.36		

t = 3.0 Sec		t = 5.0 Sec		t = 7.0 Sec		t = 10.0 Sec	
T - °F	Radius in "	T - °F	Radius in "	T - °F	Radius in "	T - °F	Radius in "
307.5	5.00	335	5.00	354	5.00	370.5	5.00
248.5	5.10	282.5	5.10	307	5.10	329.5	5.10
199.9	5.20	237.3	5.20	266.3	5.20	293.7	5.20
163.5	5.30	195.5	5.30	227.7	5.30	255.0	5.30
137.7	5.40	169.8	5.40	197.0	5.40	225.0	5.40
120.8	5.50	146.7	5.50	170.0	5.50	198.8	5.50
110.9	5.60	130.4	5.60	151.2	5.60	175.9	5.60
105.3	5.70	119.3	5.70	134.7	5.70	157.6	5.70
101.7	5.80	110.0	5.80	124.9	5.80	144.8	5.80
100.7	5.90	105.4	5.90	115.1	5.90	132.0	5.90
100.2	6.00	103.3	6.00	107.8	6.00	124.5	6.00

Figures (C) and (D) - (Numerical Method - Surface Temp.)

$$T_0' = \frac{N}{2} T_a + \left(\frac{1}{4} - \frac{N+1}{2}\right) T_0 + \frac{1}{2} T_1 \quad \text{(NOTE } M=4\text{)}$$

$\Delta x = .09$, $\Delta t = .101$, $T_0 = 100$, $T_a = 500$, $K = .50$

Time in Sec	H = 900	H = 1000	H = 1100	H = 1200
0	100	100	100	100
.101	154	160	166	172
.202	174	181	188	195
.303	188	196	204	211
.404	199	208	216	224
.505	208	217	226	235
.606	216	226	235	244
.707	223	233	242	252
.808	230	240	249	259
.909	235	246	255	265

Figure (E) - (Analytic solution - Temperature times radius vs. radius)

Radius in Inches	t = 1.0 Sec T x r	t = 3.0 Sec T x r	t = 5.0 Sec T x r	t = 7.0 Sec T x r	t = 10.0 Sec T x r
5.00	738.0	1038	1175	1270	1352
5.10	392.0	758	931	1055	1170
5.20	177.0	528	741	865	1008
5.30	64.0	337	527	676	821
5.40	19.5	203	377	524	675
5.50	6.5	114	257	385	544
5.60		61.0	170	287	425
5.70		30.2	110	198	328
5.80		9.8	58.0	144	260
5.90		4.1	31.9	89.0	188
6.00		1.2	19.8	46.8	147

Figure (G) - (Analytic Solution - Surface stress vs. time)

Surface stress in lbs/in^2	t=1.0 Sec	t=3.0Sec	t=5.0 Sec	t=7.0 Sec	t=10.0 Sec
	-38,950	-49,700	-50,250	-49,800	-47,800

Figure (F) - (Analytic Solution - Stress distribution at time (5.0 sec) of maximum stress)

Radius in Inches	σ_θ lbs/in^2	σ_r lbs/in^2	σ_z lbs/in^2
5.00	50,250	0.0	50,250
5.10	33,700	822	34,522
5.20	19,630	1,192	20,822
5.30	6,810	1,530	8,340
5.40	826	1,573	747
5.50	7,650	1,451	6,199
5.60	12,330	1,243	11,087
5.70	15,550	965	14,585
5.80	17,900	659	17,241
5.90	18,900	333	18,567
6.00	19,210	0.0	19,210

BIBLIOGRAPHY

1. Boelter, L. M. K.

   Heat Transfer Notes. University of California Press. (Revised) 1948.

2. Carslow, H. S. and J. C. Jaeger.

   Conduction of Heat in Solids. (Eng.) Clarendon Press. 1947.

3. Carslow, H. S.

   Mathematical Theory of the Conduction of Heat in Solids. Second Edition (Reprint). 1921.

4. Devore, A.

   Evaluating Film Coefficients for Single Condensing Vapors. Chemical Engineering. 58:138-139.

5. Dusinberre, G.

   Numerical Analysis of Heat Flow. First Edition. New York. McGraw-Hill. 1949.

6. Ingersoll and Zobel.

   Heat Conduction. First Edition. 1948.

7. Jacob, Max and George A. Hawkins.

   Elements of Heat Transfer and Insulation. John Wiley and Sons. Second Edition. 1950.

8. Jacob, Max.

Heat Transfer. Volume I. John Wiley and Sons. 1949.

9. Kent, Clarence H.

Transactions ASME. Applied Mechanics APM 53-13. Sept-Dec 1931, APM 54-18. Sept 30, 1932.

10. Knox, Robert J.

An Analysis of Temperature Distribution in Metal Hollow Circular Cylinders of Finite Radius Under Unsteady State Conditions from E.E.S. Data. Journal of the American Society of Naval Engineers. 62:587-599.

11. McAdams, William H.

Heat Transmission. Second Edition. McGraw-Hill. 1942.

12. Peck, R. E. and W. A. Reddie.

Heat Transfer Coefficients for Vapors Condensing on Horizontal Tubes. Industrial and Engineering Chemistry. 43:2926-2931.

13. Timoshenko, S. and J. N. Godier.

Theory of Elasticity. Second Edition. McGraw-Hill. 1951.

OCT 2 BINDERY
SEP 8 319
NOV 9 318
FEB 8 318
APR 15 318
APR 23 RENEWED
MAY 7 RENEWED

JE 5 58 18062

Thesis T227
Teepe
Transient thermal stresses in hollow circular cylinders.

OCT 2 BINDERY
SEP 8 15 Sept 319
OCT 1 OCT 16 '53 RENEWED
NOV 2 RENEWED
NOV 9 318
FEB 8 318
APR 15 318
APR 23 RENEWED
MAY 7 RENEWED
DEC 7 INTERLIB
Lewis Flight Propuls-

18062

Thesis T227
Teepe
Transient thermal stresses in hollow circular cylinders.

Library
U. S. Naval Postgraduate School
Monterey, California

thesT227
Transient thermal stresses in hollow cir
3 2768 002 03423 3
DUDLEY KNOX LIBRARY